Midterm 1 - Spring 2017-Solution

Physics 1A, Dr. Mostafa El Alaoui

Monday, April 24

Name: _____ Student I.D. #_____

Signature: _____

Please do the following 4 problems. Show all work and reasoning. Use the back of the page if necessary and circle your final answer. **Write your name and student ID on your exam.**

Problem	Points
1	
2	
3	
4	
TOTAL	

Constant: $g=9.80 m/s^2$

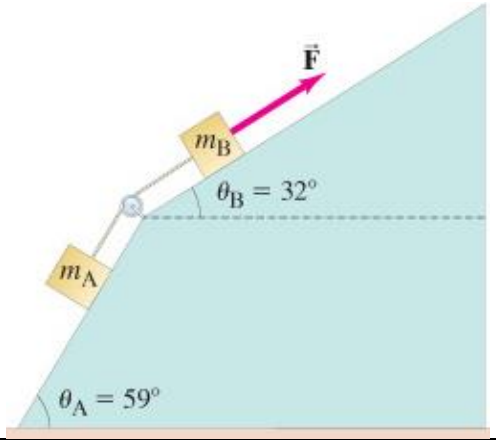
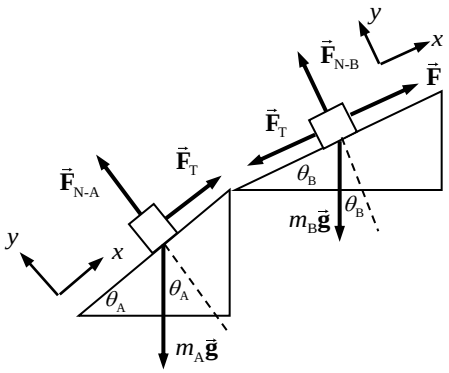
Name: _____

	Problem 1
	A golfer tries to hit the ball over a ravine, but the golf ball lands instead at the bottom of the ravine. It lands 90.0 m horizontally from its initial position and 15.0 m below it. The ball's initial velocity was directed 60.0 degrees above the horizontal. The acceleration of gravity is 9.8 m/s^2.
(10 Points)	a) What is the ball's initial velocity in the *y* direction? $x = v_0 \cos\theta_i t \quad \Rightarrow \quad t = \dfrac{x}{v_0 \cos\theta_i}$ $y = v_0 \sin\theta_i t - \frac{1}{2} g t^2 = v_0 \sin\theta_i \left(\dfrac{x}{v_0 \cos\theta_i}\right) - \frac{1}{2} g \left(\dfrac{x}{v_0 \cos\theta_i}\right)^2$ $y = x \tan\theta_i - \frac{1}{2} g \left(\dfrac{x}{v_0 \cos\theta_i}\right)^2 \Leftrightarrow \frac{1}{2} g \left(\dfrac{x}{v_0 \cos\theta_i}\right)^2 = x \tan\theta_i - y$ $v_0 = \dfrac{x}{\cos\theta} \sqrt{\dfrac{g}{2(x \tan\theta_i - y)}} = \dfrac{90.0m}{\cos 60^\circ} \sqrt{\dfrac{(9.80 m/s^2)}{2((90.0m)\tan 60^\circ + (15.0m))}} = 30.5 m/s$ $v_{0y} = v_0 \sin\theta_i = (30.5\text{m/s})(\sin(60°)) = 26.4\text{ m/s}$
(5 Points)	b) What is the maximum height the ball reaches (above its starting point)? Time of maximum height at time t_m satisfying $v_y = v_{y0} - g t_m = 0 \;\; t_m = v_{y0} / g$ t_m = (26.4m/s)/(9.80 m/s^2)=2.69s $y = v_{y0} t_m - \frac{1}{2} g t_m^2$ y = (26.4m/s)(2.69s) - (1/2)(9.80m/s^2)(2.69s)^2 y = 35.6 m
(5 Points)	c) What is the speed and angle with the horizontal (angles below the horizontal count as negative) with which the ball strikes the ground at the bottom of the ravine? At the time the ball strikes the stick $v_x = v_{x0} = v_0 \cos\theta$ $v_x = (30.5\text{m/s})(\cos(60°)) = 15.2\text{ m/s}$ The equations below can be used to determine v_y at the time the ball strikes the stick: $-2g(y - y_0) = v_y^2 - v_{y0}^2$ or $v_y^2 = -2g(y - y_0) + v_{y0}^2$ v_y^2 = (26.4m/s^2)^2 - 2(9.8m/s^2)(-15.0m-0) = 990m^2/s^2

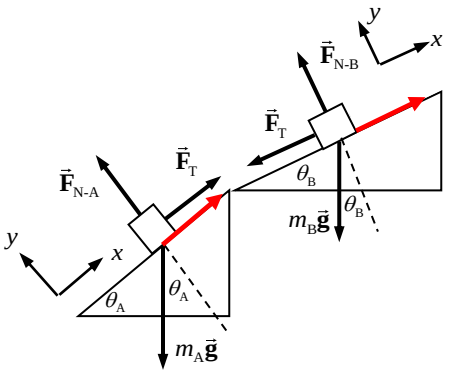
Name: _____

	v_y = -31.5 m/s (must be downward) $v = \sqrt{v_x^2 + v_v^2} = \sqrt{(15.2\text{m/s})^2 + 990\text{m}^2/s^2}$ **v = 34.9 m/s** $\theta = \arctan(v_y / v_x)$ θ = arctan((-31.5m/s)/15.2m/s)) **θ= -64.2°**
(5 Points)	d) The far edge of the ravine was 92.0 m away from the golfer's position. To just reach the far edge of the ravine, at the same initial speed, what is the angle with the horizontal that the golf ball should be launched? $y = v_0 \sin\theta_i t - \frac{1}{2} g t^2 = 0 \Rightarrow \begin{cases} t = 0 \\ t = \dfrac{2v_0 \sin\theta_i}{g} \end{cases}$ $x = \dfrac{2v_0 \sin\theta_i}{g} v_0 \cos\theta_i t \quad \Rightarrow \quad \text{x} = \dfrac{v_o^2 \sin 2\theta_i}{g} \Rightarrow \sin 2\theta_i = \dfrac{xg}{v_o^2}$ $\theta_i = \frac{1}{2} a\sin^{-1}\left(\dfrac{xg}{v_o^2}\right) = \frac{1}{2} a\sin^{-1}\left(\dfrac{(92.0m/s)(9.80m/s^2)}{(30.5m/s)^2}\right) \Rightarrow$ **$\theta_i = 37.9^o$**

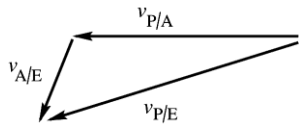
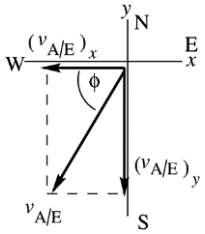
Name: _____

	Problem 2
	Consider the system shown in the figure with m_A = 9.5 kg and m_B = 11.5 kg. The angles θ_A = 59° and θ_B = 32°. 
(7 points)	a) In the absence of friction, what magnitude of force $\vec{F}$ would be required to pull the masses at a constant velocity up the fixed inclines?  We use the free-body diagram to find the force needed to pull the masses at a constant velocity. We choose the "up the plane" direction as the positive direction for both masses. Then they both have the same acceleration even if it is non-zero. $$m_A: \ \sum F_x = F_T - m_A g \sin\theta_A = m_A a = 0$$ $$m_b: \ \sum F_x = F - F_T - m_B g \sin\theta_B = m_B a = 0$$ Add the equations to eliminate the tension force and solve for F. $$\left(F_T - m_A g \sin\theta_A\right) + \left(F - F_T - m_B g \sin\theta_B\right) = 0 \ \rightarrow$$ $$F = g\left(m_A \sin\theta_A + m_B \sin\theta_B\right)$$ $$= \left(9.80\,\text{m/s}^2\right)\left[(9.5\,\text{kg})\sin 59° + (11.5\,\text{kg})\sin 32°\right] = \boxed{1.40\times10^2\,\text{N}}$$
(6 points)	b) The force $\vec{F}$ is now removed. What is the magnitude and the direction of the acceleration of the two blocks? Use the free-body diagram from above but ignore the applied force $\vec{F}$. $m_A: \ \sum F_x = F_T - m_A g \sin\theta_A = m_A a$; $m_b: \ \sum F_x = -F_T - m_B g \sin\theta_B = m_B a$ Again add the two equations to eliminate the tension force.

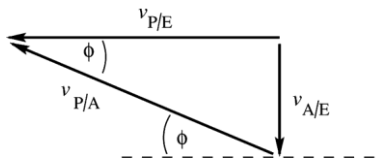
Name: _____

	$(F_T - m_A g\sin\theta_A) + (-F_T - m_B g\sin\theta_B) = m_A a + m_B a \;\rightarrow$ $a = -g\dfrac{m_A\sin\theta_A + m_B\sin\theta_B}{m_A + m_B} = -(9.80\,\text{m/s}^2)\dfrac{(9.5\,\text{kg})\sin 59° + (11.5\,\text{kg})\sin 32°}{21.0\,\text{kg}}$ $= -6.644\,\text{m/s}^2 \approx \boxed{6.64\,\text{m/s}^2\text{, down the planes}}$
(6 points)	c) In the absence of force $\vec{\mathbf{F}}$, what is the tension in the string? Use one of the Newton's second law expressions from part (*b*) to find the string tension. It must be positive if there is a tension. $F_T - m_A g\sin\theta_A = m_A a \;\rightarrow$ $F_T = m_A(g\sin\theta_A + a) = (9.5\,\text{kg})\left[(9.80\,\text{m/s}^2)(\sin 59°) - 6.644\,\text{m/s}^2\right] = \boxed{17\,\text{N}}$
(6 points)	d) In the absence of force $\vec{\mathbf{F}}$ what is the minimum coefficient of static friction applied to both block such that they remain stationary?  Along x object B : $f_{sB} - F_T - m_B g\sin\theta_B = 0$ Along y object B : $F_{N-B} = m_B g\cos\theta_B \Rightarrow f_{sB} = \mu_s m_B g\cos\theta_B$ Along x object A : $f_{sA} + F_T - m_A g\sin\theta_A = 0$ Along y object A : $F_{N-A} = m_A g\cos\theta_A \Rightarrow f_{sA} = \mu_s m_A g\cos\theta_A$ $\mu_s m_B g\cos\theta_B - F_T - m_B g\sin\theta_B + \mu_s m_A g\cos\theta_A + F_T - m_A g\sin\theta_A = 0$ $\mu_s(m_B\cos\theta_B + m_A\cos\theta_A) = (m_B\sin\theta_B + m_A\sin\theta_A)$ $\mu_s = \dfrac{m_B\sin\theta_B + m_A\sin\theta_A}{m_B\cos\theta_B + m_A\cos\theta_A}$ $\mu_s = \dfrac{(11.5kg)\sin 32^o + (9.5kg)\sin 59^o}{(11.5kg)\cos 32^o + (9.5kg)\cos 59^o}$ $\mu_s = 0.972$

Name: _____

	Problem 3
	An airplane pilot sets a compass course due west and maintains an airspeed of 220 km/h. After flying for 0.500 h, she finds herself over a town 120 km west and 20 km south of her starting point.
(10 points)	a) What is the wind velocity (magnitude and direction)? Select a coordinate system where $+y$ is north and $+x$ is east. $\vec{v}_{P/E}$, the velocity of the plane relative to the earth. $\vec{v}_{P/A}$, the velocity of the plane relative to the air (the magnitude $v_{P/A}$ is the airspeed of the plane and the direction of $\vec{v}_{P/A}$ is the compass course set by the pilot). $\vec{v}_{A/E}$, the velocity of the air relative to the earth (the wind velocity). $$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}.$$ $\vec{v}_{P/A}$ has magnitude 220 km/h and its direction is west. So $(v_{P/A})_x = -220 \text{ km/h}$ and $(v_{P/A})_y = 0.$ From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{P/E}$ has components in our coordinate system of $$(v_{P/E})_x = -\frac{120 \text{ km}}{0.500 \text{ h}} = -240 \text{ km/h} \quad \text{(west)}$$ $$(v_{P/E})_y = -\frac{20 \text{ km}}{0.500 \text{ h}} = -40 \text{ km/h} \quad \text{(south)}$$ With this information the diagram corresponding to the velocity addition equation is shown in  We are asked to find $\vec{v}_{A/E}$, so solve for this vector: $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ gives $\vec{v}_{A/E} = \vec{v}_{P/E} - \vec{v}_{P/A}.$ The *x*-component of this equation give $(v_{A/E})_x = (v_{P/E})_x - (v_{P/A})_x = -240 \text{ km/h} - (-220 \text{ km/h}) = -20 \text{ km/h}.$ The *y*-component of this equation gives $(v_{A/E})_y = (v_{P/E})_y - (v_{P/A})_y = -40 \text{ km/h}.$ $\vec{v}_{A/E}$ we can find its magnitude and direction. $v_{A/E} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2}$  $v_{A/E} = \sqrt{(-20 \text{ km/h})^2 + (-40 \text{ km/h})^2} = 44.7 \text{ km/h}$ $\tan\phi = \frac{40 \text{ km/h}}{20 \text{ km/h}} = 2.00; \quad \phi = 63.4°$ The direction of the wind velocity is $63.4°$ S of W, or $26.6°$ W of S.

Name: _____

(10 points)	b) If the wind velocity were 40 km/h due south, in what direction should the pilot set her course to travel due west? Use the same airspeed of 220 km/h The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a). $\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found. $\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south. The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given. The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.71c.  **Figure 3.71c** The vector addition diagram forms a right triangle. **EXECUTE:** $\sin\phi = \frac{v_{A/E}}{v_{P/A}} = \frac{40 \text{ km/h}}{220 \text{ km/h}} = 0.1818;$ $\phi = 10.5°.$ The pilot should set her course $10.5°$ north of west.

Name: _____

	Problem 4
	In 100 m race, it took both Maggie and Judy 10.4 s to cross the finish line. Each one accelerated at a uniform rate until she reached her final speed. Maggie took 2.00 s and Judy 3.00 s to attain their maximum speeds which they maintained for the rest of the race.
(10 points)	a) What was the acceleration of each sprinter? The total distance (displacement) that each runner covers is the same: $\Delta x = \Delta x_a + \Delta x_v$ $\Delta x = \frac{1}{2}a\Delta t^2 + a\Delta t\,(T - \Delta t)$ $\Delta x = a[\frac{1}{2}\Delta t^2 + \Delta t\,(T - \Delta t)]$ so $$a = \frac{\Delta x}{\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)}$$ where Δx = 100 m, and T = 10.4 s. For Maggie (runner 1), Δt_1 = 2.00 s a_1 = (100 m)/(18.8 s^2) = 5.32 m/ s^2 For Judy (runner 2), Δt_2 = 3.00 s: a_2 = (100 m)/(26.7 s^2) = 3.75 m/ s^2
(6 points)	b) What were their maximum speeds? Maggie (runner 1): $v_1 = a_1\Delta t_1$ = 10.6 m/ s Judy (runner 2): $v_2 = a_2\Delta t_2$ = 11.2 m/ s
(9 points)	c) What is the maximum distance by which Judie was behind Maggie, and at what time did that occur? Maggie accelerates at the greater rate, so she will be ahead of Judie at, and immediately after, the 2.00-s mark. After the 3.00-s mark, Judie is travelling faster than Maggie, so the distance between them will shrink. In the time interval from the 2.00-s mark to the 3.00-s mark, the distance between them will be the greatest. During that time interval, the distance between them (the position of Maggie relative to Judie) is $D = \Delta x_1 - \Delta x_2 = \frac{1}{2}a_1\Delta t_1^2 + a_1\Delta t_1(t - \Delta t_1) - \frac{1}{2}a_2 t^2$

Name: _____

	$\frac{dD}{dt} = a_1\Delta t_1 - a_2 t = 0$ $t = \frac{a_1}{a_2}\Delta t_1 = \frac{(5.32m/s^2)}{(3.75m/s^2)}(2.00s) = 2.84s$ $D = \frac{1}{2}\left(\frac{5.32m}{s^2}\right)(2.00s)^2 + \left(\frac{3.75m}{s^2}\right)(2.00s)(2.84s - 2.00s)$ $-\frac{1}{2}\left(\frac{5.32m}{s^2}\right)(2.84s)^2$ $D = 4.47m$ Maggie ahead of Judie by 4.47 m.